New Methods For Predictability Analysis

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LONG-TERM GOAL

Our long-term goal is to improve the accuracy of atmospheric and oceanic deterministic and statistical forecast. Specifically, we seek to maximize deterministic forecast accuracy while minimizing observational and computational cost. We also seek to maximize the accuracy and utility of statistical forecasts, which can provide forecast information beyond the time interval over which deterministic forecast is possible.

OBJECTIVES

Our present objective is to use advances in fundamental understanding of deterministic and statistical forecast dynamics to develop practical methods for improving forecast. In particular we exploit model order reduction techiques derived from control theory together with the formal equivalence between 4dVar and the extended Kalman filter to implement an approximate optimal state estimation for forecast initialization. We are also developing statistical methods for improving the accuracy of ensemble forecasts by accounting for model uncertainty in the choice of ensembles. We intend to improve the accuracy of forecast of atmospheric statistics including the mean strength and preferred location of cyclones and their heat and momentum transports by identifying the sensitivity of these quantities to changes in boundary conditions such as sea surface temperature.

APPROACH

We employ both analytical and numerical methods primarily derived from the theory of dynamical systems, the theory of stochastic differential equations and control theory to study error growth in deterministic forecast. In addition we use methods drawn from statistical analysis of certain and uncertain systems to address questions of the statistical predictability of weather and climate which become applicable for times exceeding the time interval over which deterministic forecast is possible. Our approach is first to improve fundamental understanding of error dynamics and then use this theoretical base to develop practical methods for computing error statistics so that advanced forecast methods such as optimal state estimation algorithms can be effectively implemented. One approach we are pursuing is to reduce the dimension of the error system so that an approximate Kalman filter can be implemented. With the objective of practical application we are particularly interested in developing algorithms for optimal state estimation that utilize forecast products and algorithms such as

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14. ABSTRACT
Our long-term goal is to improve the accuracy of atmospheric and oceanic deterministic and statistical forecast. Specifically, we seek to maximize deterministic forecast accuracy while minimizing observational and computational cost. We also seek to maximize the accuracy and utility of statistical forecasts, which can provide forecast information beyond the time interval over which deterministic forecast is possible.

15. SUBJECT TERMS					

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the adjoint integrator that is already available at many forecast centers for use in operational variational data assimilation.

The foundation of the approach we use is provided by recent theoretical advances in non-normal time-dependent stability analysis. We use these advances together with methods of modern control theory, specifically balanced Hankel operator truncation methods, to reduce the dimension of the error system. Having reduced the order of the error system we then use this reduced order system to obtain an effective Kalman gain for state identification in the full forecast system. Implementing these methods in operational forecast environments requires modifying theoretically optimal methods to use available resources to best advantage.

We are developing theory and methods for incorporating the effects of uncertainty in the forecast error system. Uncertainty can arise in the forecast system both from incomplete knowledge of the forecast trajectory and from parameterized physics. Ensemble forecast is improved by accounting for model uncertainty in forecast error growth which differs fundamentally from the more familiar uncertainties associated with initial conditions. We apply recent theoretical advances in non-normal time-dependent stability analysis to the problem of error growth in uncertain forecast systems. Optimal excitation theory is then used to construct optimal ensembles.

We are developing theory for determining the sensitivity of atmospheric statistics to structured changes in the mean jets and boundary conditions. The method used builds on previous work in which storm track statistics were obtained using a stochastic model of jet turbulence. These results extend generalized stability theory to address statistical stability of jets.

The above described work is a joint effort between Professors Brian Farrell and Petros Ioannou.

WORK COMPLETED

We have completed work on the fundamental theory for error growth in time dependent systems.

We have completed the theory for optimal reduction of the error system order based on balanced truncation of the error dynamics and have shown that error dynamics in these reduced equations accurately models time dependent forecast system error.

We have adapted optimal balanced Hankel operator model order reduction previously used in association with discrete time independent engineering systems to the time dependent forecast error dynamical system.

We have constructed a reduced order Kalman filter using balanced truncation and demonstrated its ability to accurately estimate the state of the model forecast.

We have developed the theory of error growth in uncertain forecast systems, obtained the error growth and structure under the influence of statistically distributed parameterizations, and obtained the statistically optimal structure for use in ensemble generation.

RESULTS

We have developed a stability theory for time dependent systems that allows better prediction of forecast error growth. We have shown that the dominant error growth arises from destabilization of a restricted set of non-normal vectors of the mean operator by time dependence (Farrell and Ioannou, 1999). These results allow us to reduce the dimension of the unstable dynamics of time dependent tangent linear error systems by representing the dynamics in a restricted subspace.

We have developed a method for reducing the dimension of the time independent error system based on retaining the dominant error subspace. This method truncates the error growth dynamics in balanced coordinates (Moore, 1981; Glover, 1984; Farrell and Ioannou, 2001a).

We have implemented and tested the balanced truncation method in model problems. We have found in our model problems that a very good approximation to the error dynamics is obtained with dimension reduction by as much as a factor of 10 (Farrell and Ioannou, 2001a). Preliminary work at ECMWF suggests that propagation of error covariance of the forecast system using this method can be accurately achieved by retaining $O(10^3)$ degrees of freedom.

We have determined that truncating the dynamical system in a balanced realization of the optimals and evolved optimals for a single appropriately chosen time provides a nearly optimal reduced order error system. This is a significant result because it suggests that implementing the order reduction algorithm in an operational forecast mode can be greatly simplified (Farrell and Ioannou, 2001a).

We have extended the balanced truncation method of optimal order reduction to a time dependent Lyapunov unstable error system (Farrell and Ioannou, 2001b).

We have obtained a reduced order Kalman filter in a model time dependent tangent linear forecast system. We showed that this reduced order Kalman filter successfully observes a model of the atmospheric error state (Farrell and Ioannou, 2001b).

We have developed the theory of error dynamics in uncertain systems for application to the ensemble forecast problem and obtained exact dynamical equations for the evolution of an ensemble mean field and the evolution of the ensemble covariance under uncertain dynamics (Farrell and Ioannou, 2002a and 2002b).

We have solved the problem of optimal excitation of uncertain systems. We first proved that the optimal excitation problem for uncertain systems has a solution: in uncertain systems there is a sure initial condition producing the greatest expected perturbation growth and a sure structure that is most effective in exciting variance when this structure is continuously forced. We then obtained a practical algorithm for finding the statistical optimals (Farrell and Ioannou, 2002c). These results are being used to develop a method for optimal ensemble generation.
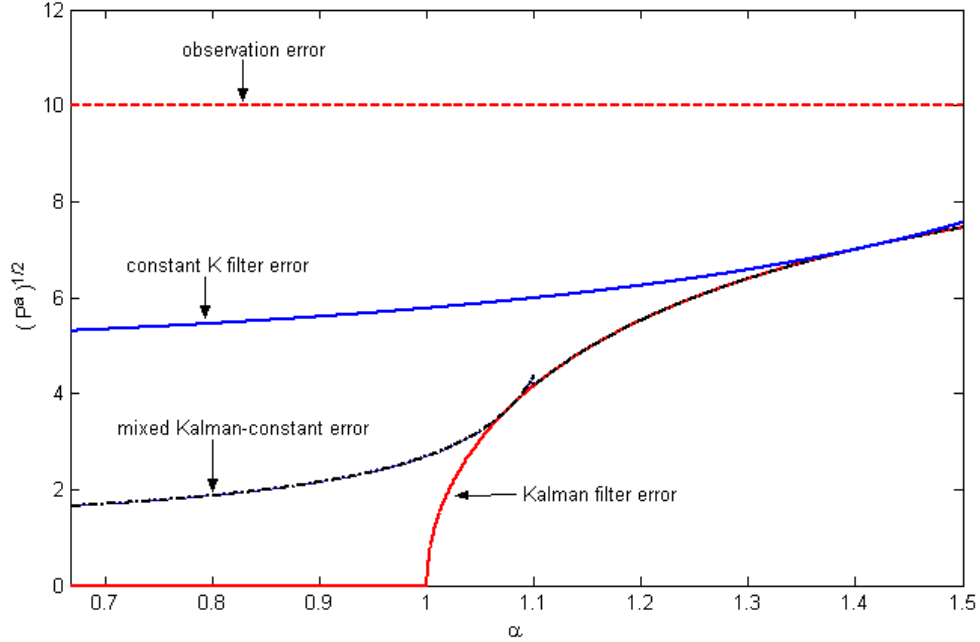
Comparison of the performance of the Kalman filters, the approximate Kalman filter, and presently used methods of data assimilation. Shown is the r.m.s. forecast analysis error, $(P^a)^{1/2}$ (m), as a function of error growth, α, over an assimilation period for the various methods of data assimilation. The red dot curve is the observational r.m.s. error, $R^{1/2}$. The red line is the analysis error obtained by the statistically optimal Kalman filter with gain $K_{opt} = \alpha^2 P^a/(\alpha^2 P^a + R)$. The blue curve is the analysis error for assimilation with constant gain $K = B/(B+R)$, where B, the background error covariance, has been selected to stabilize the fastest growing mode with growth $\alpha_m = 1.5$ with some margin $(B = 1.25(\alpha_m-1)R)$. This method approximates the 4 dimensional variational data assimilation method that is presently used in operational forecast. The black curve is the analysis error variance that results from employing the Kalman filter on the modes with growths greater than $\alpha_r = 1.1$ and a constant gain for the remaining modes, simulating the effect of the reduced Kalman filter. The reduced Kalman filter results in a substantial reduction in analysis error.

IMPACT/APPLICATION

Our results are presently being used to implement advanced methods of state estimation in forecast models including approximations to the Kalman filter. Our method for accounting for model uncertainty in ensemble generation is directly applicable to improving ensemble forecast methods currently being used.

TRANSITIONS

Our method for obtaining more accurate initial conditions has been configured to use operational forecast products and is presently being implemented in an operational forecast at ECMWF.

RELATED PROJECTS

None

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